Choice E is NOTA, meaning 'none of the above answers are correct'. Let $f^{(n)}(x)$ denote the n^{th} derivative of $f(x)$ where defined. For number 29, it may be helpful to find the derivative of $\sinh^{-1}(x)$ then proceed. Be sure to read the questions carefully and have fun!

1) Use a Left-Hand Riemann sum approximation to find the area under the curve $y=\ln(x)$ from $x=1$ to $x=13$ using 4 subdivisions of equal length.

A. $\ln(45)$ B. $3\ln(280)$ C. $\frac{3}{2}\ln(280)$ D. $3\ln(3640)$ E. NOTA

2) If $a(x)=\cos\left(x+\frac{\pi}{2}\right)$, what is $a''(x)$?

A. $\sin(x)$ B. $\cos(x)$ C. $-\sin(x)$ D. $-\cos(x)$ E. NOTA

3) Which of the following functions are differentiable on the domain $[-1,1]$?

I. $\sec^3(x)$ II. $\ln|x|$ III. $\cos(x)-x^3$ IV. $\frac{x}{\cos(x)}$

A. II, III, and IV B. I, II and IV C. I, III, and IV D. I and III E. NOTA

4) Which of the following limits exist and are finite?

A. $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$

B. $\lim_{x\to-\infty}\left(\frac{\sqrt{x^2-4}}{1-2x}\right)$

C. $\lim_{x\to\infty}\left(\frac{x^4-9x^2+3x}{x^3+x^2-x+1}\right)$

D. More than one of the above

E. NOTA

5) Frank consistently brags about his calculus knowledge. He tells Clare that the value of c that satisfies the Mean Value Theorem for derivatives on the interval $[-2,4]$ for the function $b(x)=\frac{1}{3}x^2-\frac{4}{3}x$ is…

A. 2 B. 1 C. 0 D. -1 E. NOTA

6) Clare, however, is pretty good herself. She challenges Frank to find the largest value of c that satisfies the Mean Value Theorem for integrals on the interval $[-2,4]$ for the function $b(x)=\frac{1}{3}x^2-\frac{4}{3}x$. He is not stumbled! He finds the answer to be…

A. 0 B. 1 C. 2 D. 4 E. NOTA

7) Find the area bound by the curve $2y|x|+|x|=4$ and the x-axis from $x=-4$ to $x=-1$.

A. $\ln(16)+\frac{3}{2}$ B. $\ln(16)-\frac{3}{2}$ C. $-\ln(16)+\frac{3}{2}$ D. $\ln(256)+3$ E. NOTA

8) Find $\frac{dy}{dx}$ at the point (0,1), where $2xy+e^y\cos(x)=e$.

A. $-e^{-1}$ B. 0 C. $\frac{e}{2}$ D. $-2e^{-1}$ E. NOTA

9) Find the slope of the normal line to the function $g(x)=\sqrt{x^2+7}$ at $x=3$.

A. $-\frac{4}{3}$ B. $-\frac{3}{4}$ C. $\frac{3}{4}$ D. $\frac{4}{3}$ E. NOTA

10) What shape is formed when a vertical or horizontal ellipse centered at the origin is revolved around the x-axis?

A. Ellipsoid B. Oval C. Cardioid D. Super Ellipse E. NOTA

11) Evaluate: $\lim_{x\to0}\left(\frac{x}{\sqrt{x+1}-1}\right)$

A. 0 B. 1 C. 2 D. ∞ E. NOTA

12) For a hypothetical function $f(x)$ that is continuous, monotonically decreasing, and concave down on the domain of all reals, which of the following must be true?

A. $\int_{-a}^{0} f(x)\,dx = -\int_{0}^{a} f(x)\,dx$

B. $f(x)$ always decreases at a decreasing rate.

C. The inverse of $f(x)$ is not a function.

D. There is exactly one saddle point at the point where $f(x)$ crosses the x-axis.

E. NOTA

13) Find the area of the region bound by the graphs $f(x)=e^{2x}$, $h(x)=e^{-x}$ and the x-axis.

A. 1 B. $\frac{3}{2}$ C. 2 D. 3 E. NOTA

14) Use three iterations of Newton's Method to approximate the positive zero of $f(x)=x^2-x-5$ using an initial guess of $x_0=4$.

A. 3 B. $\frac{321}{115}$ C. $\frac{14}{5}$ D. $\frac{54}{19}$ E. NOTA

15) What is the area bound by the graphs of $f(x)=\cos^2(x)$ and $g(x)=-\sin^2(x)$ from $x=\frac{\pi}{4}$ to $x=\frac{\pi}{3}$?

A. $\frac{\sqrt{3}-2}{4}$ B. $\frac{\sqrt{6}-\sqrt{2}}{4}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{12}$ E. NOTA

16) Evaluate the following integral: $\int_{0}^{\frac{\pi}{4}} \sin^2(x)\cos^2(x)\tan(x)\,dx$

A. $\frac{1}{4}$ B. $\frac{1}{8}$ C. $\frac{1}{16}$ D. $\frac{1}{32}$ E. NOTA

17) At an exact moment in time, a cube with side length of 5 inches has a volume that is increasing at a rate of 15 cubic inches per minute. This cube, however, is inscribed in a sphere. At what rate is the radius of the sphere increasing at this point in time? (assume appropriate units for your answer)

A. $\frac{\sqrt{3}}{10}$ B. $\frac{\sqrt{3}}{5}$ C. $\frac{125\sqrt{3}}{2}$ D. $\frac{1}{5}$ E. NOTA

18) Find the interval of convergence for the following. If you believe it is divergent, select E.

$$\sum_{n=1}^{\infty}\left(\frac{n(1-x)^{\frac{n}{3}}}{3^{n-1}}\right)$$

A. $-26<x<28$ B. $-26<x\le 28$ C. $-1<x<1$ D. $-1\le x\le 1$ E. NOTA

19) What is the slope of the tangent line to the polar curve $r=3\cos(2\theta)$ at $\theta=\frac{\pi}{6}$?

A. $-3\sqrt{3}$ B. $\frac{-3\sqrt{3}}{5}$ C. $\frac{\sqrt{3}}{5}$ D. $\frac{\sqrt{3}}{7}$ E. NOTA

20) Consider the graph of $f(x)=\frac{1}{2}x^3+3$ on the interval $[2,8]$. There exists some point (a,b) on $y=f(x)$ on [2,8] such that the slope of tangent line $y=L(x)$ to $y=f(x)$ at (a,b) is equal to the slope of the secant line of $y=f(x)$ that is drawn from $x=2$ to $x=8$. What is the abscissa of the x-intercept of $y=L(x)$?

A. $\frac{56\sqrt{7}-3}{42}$ B. $\frac{2\sqrt{7}-3}{14}$ C. $-56\sqrt{7}+3$ D. $\frac{8\sqrt{7}+3}{21}$ E. NOTA

21) Doug is good at integrals. He knows that some require clever manipulation in order to integrate. Help him with the following integral, given that the angles correspond to first quadrant angles in standard position:

$$\int \sqrt{36x^2 \sec^5\left(2x^2\right) - 36x^2 \sec^3\left(2x^2\right)}\, dx\,.$$

A. $\tan^{\frac{3}{2}}\left(x^2\right)+C$ B. $\sec^{\frac{5}{2}}\left(2x^2\right)+C$ C. $\sec^{\frac{3}{2}}\left(2x^2\right)+C$ D. $\sec^{\frac{3}{2}}\left(x^2\right)+C$ E. NOTA

22) Double integrals utilize the concepts taught in single variable calculus and apply them to functions written in terms of two or more variables. From double integrals and beyond, it is possible to find area (or volume) bound by thousands of different graphs! The caveat is that integrals are sometimes difficult to evaluate as written. In these cases, it may help to change the order of integration into something far easier to evaluate by hand using elementary calculus. To illustrate this concept, reverse the order of integration for the following double integral:

$$\int_0^{16} \int_{\frac{x}{2}}^{\sqrt{4x}} \left(\frac{1+x^2}{y^2}\right) dy\, dx$$

A. $\int_{\frac{x}{2}}^{\sqrt{4x}} \int_0^{16} \left(\frac{1+x^2}{y^2}\right) dx\, dy$ B. $\int_{\frac{x}{2}}^{\sqrt{4x}} \int_0^{16} \left(\frac{1+y^2}{x^2}\right) dx\, dy$ C. $\int_{\frac{y^2}{4}}^{2y} \int_0^{8} \left(\frac{1+x^2}{y^2}\right) dx\, dy$ D. $\int_0^{8} \int_{\frac{y^2}{4}}^{2y} \left(\frac{1+x^2}{y^2}\right) dx\, dy$

E. NOTA

23) Use two iterations of Euler's method with a step size of $h=\frac{1}{2}$ to approximate $y(2)$ given that $y'=xy+2x-2y$ and the initial condition $y(1)=0$.

A. $\frac{17}{4}$ B. $\frac{9}{4}$ C. $\frac{5}{4}$ D. 1 E. NOTA

24) Jackie and Remy were walking on a sidewalk. Along the way, Remy trips and drops his heavy briefcase in a hole in the ground created by construction crews. Reluctantly, the crew decides to help him get his briefcase. The crew sends a worker, Garrett, to the bottom of the 80-foot hole with a bucket attached to the end of a rope weighing only 1.5 pounds per foot. The other end of the rope is held by the rest of the crew at the top, and when Garrett puts the briefcase in the bucket, the bucket weighs a total of 50 pounds.

How much work (in foot-pounds) is required by the rest of the crew to pull the 50-pound bucket all the way to the top of the hole? Assume Garrett remains at the bottom.

A. 416,000 B. 208,000 C. 12,400 D. 8,800 E. NOTA

25) Which of the following statements is always true?

A. If a_n is a sequence of positive integers, then $\prod_{n=1}^{\infty}(1+a_n)$ converges if and only if $\sum_{n=1}^{\infty} a_n$ converges.

B. Let a_n and b_n be sequences such that $a_n < b_n$. By the direct comparison test, if $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges as well.

C. By the ratio test, a series $\sum a_n$ will be conditionally convergent if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=1$

D. The alternating harmonic series and the harmonic series are both divergent.

E. NOTA

26) In higher level calculus, describing the behaviors of graphs can become very difficult if restricted to a single variable. For a function, say, $z = f(x, y) = \left(2x + 4xy^3\right)^3$, the derivative can be evaluated by measuring the change in one variable and leaving the others as constants. Doing this to all variables, one at a time, is called Partial Differentiation. In addition, the notation $\frac{\partial f}{\partial x}$ denotes "The partial derivative of *f* with respect to *x*" and $\frac{\partial f}{\partial y}$ denotes "The partial derivative of *f* with respect to *y*". Using this information and your previous knowledge on derivatives, compute $\frac{\partial x}{\partial y}$ for $z = \left(2x + 4xy^3\right)^3$ where defined.

A. $3\left(2x+4xy^3\right)^2\left(2+4y^3\right)$

B. $\dfrac{12xy^2}{2+4y^3}$

C. $3\left(2x+4xy^3\right)^2\left(2+12xy^2\right)$

D. $\dfrac{2+12xy^2}{\left(2x+4xy^3\right)^2}$

E. NOTA

27) Hopefully you are well acquainted with higher order derivatives. Find $f^{(2018)}(-2018)$ if $f(x) = x^{2019} - e^{-2018x}$

A. $(2019!)2018 - 2018^{2018} e^{(2018)^2}$

B. $-2018(2019!) - 2018^{2018} e^{(-4036)}$

C. $((2018!)!)2018^{2019!-2018} - 2018^{2019} e^{(-2018)^2}$

D. $-2018(2019!) - 2018^{2019} e^{(-2018)^2}$

E. NOTA

28) Let there be a set of *n*-sided polygons such that $n \in \{3, 4, 5...\}$. If all these polygons have a respective circumradius of 6, what is the convergent area of this sequence of polygons as the number of sides get infinitely large?

In other words, apply Archimedes' Method of Exhaustion to find the limit of the area of an infinite sequence of polygons with an increasing number of sides (to infinity) with the restriction that the circumradius of each polygon is always 6.

A. 36π B. 25π C. ∞ D. 49π E. NOTA

29) Compute $\int x^2 \sinh^{-1}(x)\,dx$ given the fact that $\sinh(y) = \frac{e^y - e^{-y}}{2}$ and $\cosh(y) = \frac{e^y + e^{-y}}{2}$. For this question, you may assume this function and it's derivative to be defined in the real and complex planes, if applicable.

A. $\dfrac{x^3\cosh^{-1}(x)\sqrt{x^2+1}+\sinh^{-1}(x)}{3\sqrt{x^2+1}}+C$

B. $\dfrac{3x^3\sinh^{-1}(x)+x^2+1}{6\sqrt{x^2+1}}+C$

C. $\dfrac{3x^3\sinh^{-1}(x)-\sqrt{x^2+1}\left(x^2-2\right)}{9}+C$

D. $\dfrac{\sinh^{-1}(x)\left(x^3+x^2\right)-2}{6}+C$

E. NOTA

30) Congratulations on reaching the end of this test! For this final question, compute $f'(3)$ if $f(x) = \ln\left(\frac{i(x!)}{(x+1)!}\right)$. For this question, you may assume this function and its derivative to be defined in the real and complex planes, if applicable.

A. $-\ln(4)$ B. $-\dfrac{1}{4}$ C. 0 D. $\dfrac{1}{64}$ E. NOTA